WORK IN PROGRESS · MATHEMATICS, ENUMERATIVE COMBINATORICS

Growth of pattern-avoiding ascent sequences

MANTIS DISCOVERY

MANTIS · MIT CSAIL

OCTOBER 2026

01

The question

An ascent sequence starts at 0, and each new entry is at most one more than the number of ascents so far. How many ascent sequences of length n avoid a family of patterns such as 0120, 0121 and 0122, and how fast does that number $A(n)$ grow?

02

How we are approaching it

- Join consecutive occurrences of the same value. These arcs form a repetition graph.
- Bound its chromatic number, so that a sequence can be rebuilt from one occurrence of each value and a little extra data.
- Compare with permutations for the upper bound, and append distinct larger entries to suitable prefixes for the lower bound.

03

What we have so far

For any family of patterns $012\cdots(d{+}u)d$ that each use at least three values, $\log A(n) = n\log n - 2n\log\log n + O(n)$.

The generating functions are not D-finite, and the exponential one is entire of infinite order. With a pattern of fewer than three values, the generating function is rational.

04

Exhaustive counts

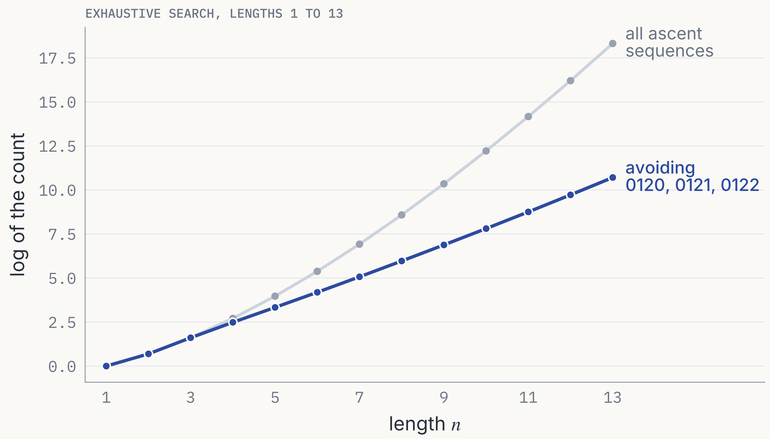


FIGURE 1. We counted by exhaustive search, up to $n = 13$, the sequences of each length that avoid 0120, 0121 and 0122 (blue) and all ascent sequences (grey). At $n = 13$, 44,616 of the 89,903,100 ascent sequences avoid the patterns.

05

A repetition graph

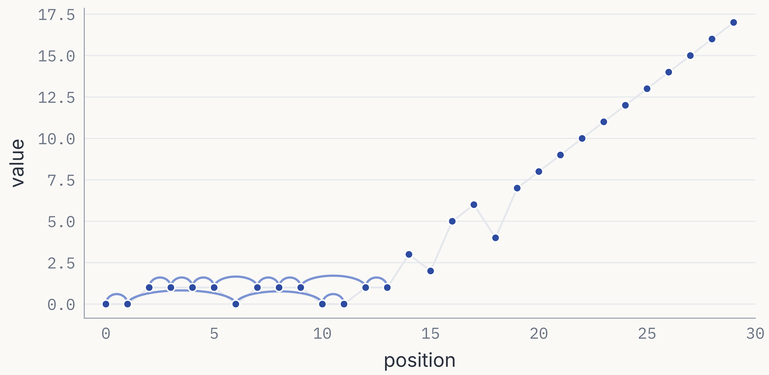
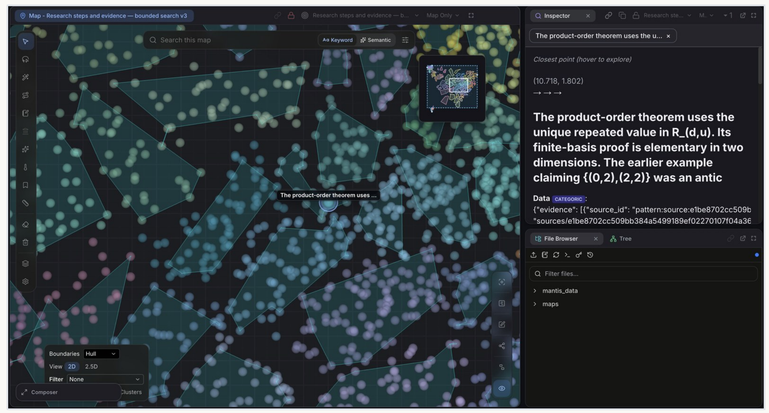


FIGURE 2. This avoiding sequence has length 30, and each arc joins consecutive occurrences of the same value.

06

In Mantis



3,230 RECORDS · THE OPEN RECORD

The product-order theorem uses the unique repeated value in $R_{(d,u)}$.

The same record corrects an earlier example: $\{(0,2),(2,2)\}$ had been claimed as an antichain, and the replacement is $\{(0,3),(2,1)\}$.

ALSO ON RECORD

Review caught an invalid weakening of an inequality.

07

Open questions

- The linear term of $\log A(n)$ is known for 0120, 0121 and 0122 avoided together. What is it for other families?
- Which other families of sequences grow like $n\log n - 2n\log\log n$?