Stability of the Gaussian log-Sobolev inequality

MANTIS DISCOVERY

MANTIS · MIT CSAIL

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01

The question

The Gaussian logarithmic Sobolev inequality bounds entropy by Fisher information, and Gaussians make it sharp. If a density nearly attains it, how close must the density be to a Gaussian, and can the constants be independent of the dimension?

02

How we are approaching it

- Study the transport first variation of the deficit: how the gap changes when the density is moved a little.
- Find its smallest possible size at fixed Fisher information and second moment, and every case where the minimum is reached.
- Integrate it along the Ornstein-Uhlenbeck flow, which carries any density to the Gaussian.

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What we have so far

We determine the smallest squared norm of the first variation at prescribed Fisher information and a second-moment cap, together with every case of equality.

Integrated along the flow, it controls Wasserstein distance, Fisher information and the deficit with sharp constants, with no covariance or Poincaré assumption. Centered isotropic Gaussians give equality.

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Densities along the flow

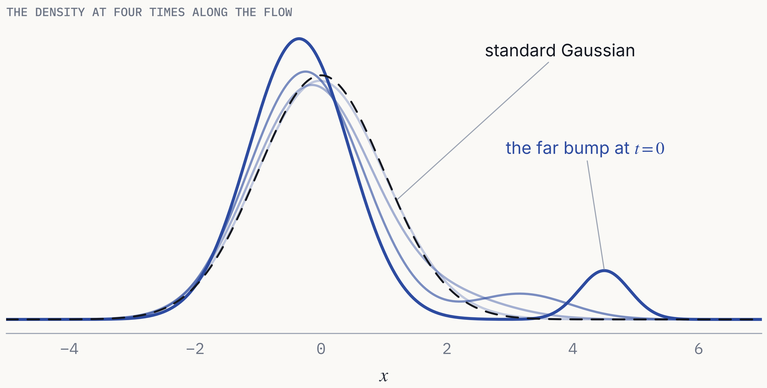


FIGURE 1. The Ornstein-Uhlenbeck flow carries a density with a small far bump toward the standard Gaussian. A mixture of two Gaussians stays a mixture under the flow, so these curves are exact.

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The deficit along the flow

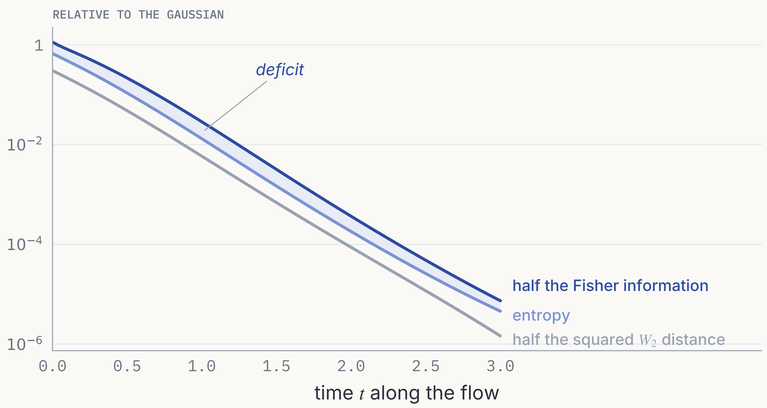
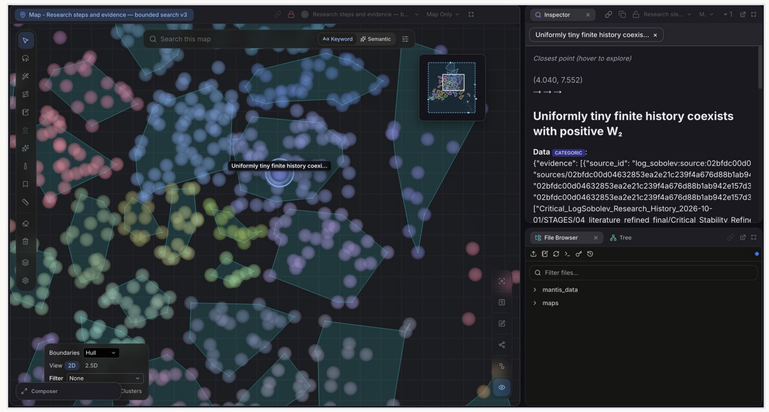


FIGURE 2. For the same density we computed half its relative Fisher information, its relative entropy and half its squared Wasserstein distance to the Gaussian. The log-Sobolev inequality keeps the entropy below half the Fisher information, and the shaded gap between them is the deficit. Talagrand's inequality keeps the distance below the entropy.

06

In Mantis



1,036 RECORDS · THE OPEN RECORD

Uniformly tiny finite history coexists with positive W_2.

The record is a counterexample proved in the manuscript: over any fixed time window the observed quantity can stay uniformly small while the Wasserstein distance to the Gaussian stays positive. So no fixed observation interval is enough.

ALSO ON RECORD

Gaussian mixtures show that dimension-free stability cannot come from the instantaneous quantity alone.

07

Open questions

- The escaping parts of minimizing sequences need not be Gaussian. What can they look like?
- The flow estimates extend to closed weighted manifolds with positive weighted Ricci curvature. How much further do they go?