WORK IN PROGRESS · COMPUTER SCIENCE, GAMES AND ALGORITHMS

Repetition under a deadline

MANTIS DISCOVERY

MANTIS · MIT CSAIL

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01

The question

Two players move a token around a graph. A player may claim a repetition, budgets expire as play goes on, and nothing resets on a directed cycle. A monitor watching the game must report its exact value at every moment. How much of the history does the monitor have to remember?

02

How we are approaching it

- Reduce a history to a draw boundary in the current strongly connected component, plus one bit for a current claim.
- Upper bound: forget counts that cannot reach the repetition threshold k before the budget runs out.
- Lower bound: find common continuations that tell apart histories of the same length.

03

What we have so far

For arenas with $N \geq 8$ vertices, no memory is needed when $H \leq 2k - 2$. Otherwise exact monitoring needs $\Theta\big(\min\{N \log k,\ \log \binom{N+S}{S}\}\big)$ bits; the cases $k = 2$, $H \in \{3, 4\}$ need exactly one bit.

Randomization with error below $1/2$ does not beat the dense $\Theta(N \log k)$ bound.

04

Memory needed

FIGURE 1. The curves show the order of the memory needed as the capacity H grows, for $N = 8$ vertices and three thresholds k. None is needed up to $H = 2k - 2$ (dotted), and after that it takes $\min\{N \log k,\ \log \binom{N+S}{S}\}$ bits with $S = H - 2k + 2$. Constants are left out.

05

Memory over capacity and threshold

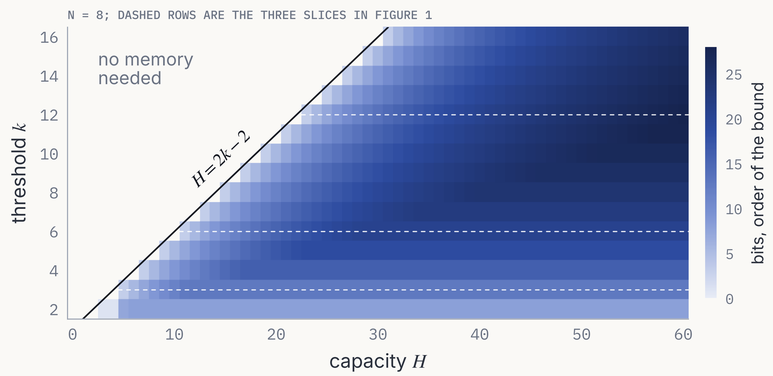
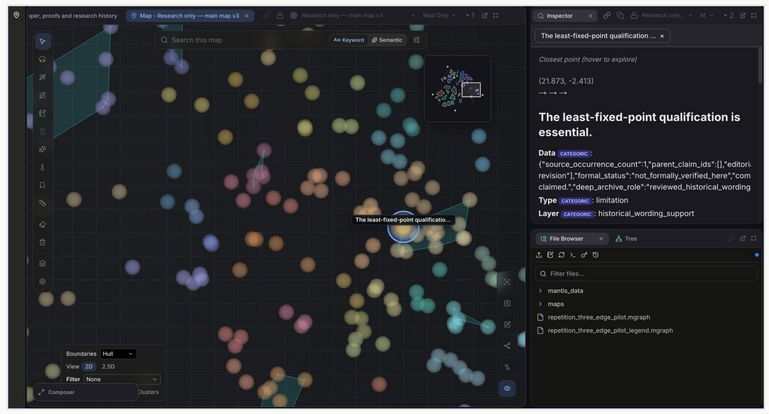


FIGURE 2. This map shows the same bound over capacity H and threshold k, for $N = 8$. Below the line $H = 2k - 2$ no memory is needed, and the dashed rows are the three thresholds of Figure 1.

06

In Mantis



617 RECORDS · THE OPEN RECORD

The least-fixed-point qualification is essential.

The record is a counterexample from the manuscript: with $H = 4$, starting from $(q, 1)$, no positional choice wins, but resetting once does.

ALSO ON RECORD

The classification in N, H and k replaced an earlier worst-case $\Theta(N \log k)$ memory theorem.

For clock games with positive costs, all capacity profiles together have at most N breakpoints and $N(N+1)/2$ constant runs, and the quadratic count is sharp.

07

Open questions

- The memory bounds are proved for arenas with at least eight vertices. What happens on smaller ones?
- The history results reach infinite arenas with a well-founded potential for resets. Which other infinite arenas do they cover?