Sidon subsets through random rounding

MANTIS DISCOVERY

MANTIS · MIT CSAIL

OCTOBER 2026

01

The question

A Sidon set is a set of numbers whose pairwise sums $a+b$ are all different. Any set A of N numbers contains a Sidon subset. How large a subset can we always find, whatever A is? Write

$$\ell(N) = \min_{|A|=N} \max\{\,|S| : S \subseteq A \text{ is Sidon}\,\}.$$

Lower bounds of the form $\ell(N) \geq c\sqrt{N}$ are known. We want to know how large the constant c can be.

02

How we are approaching it

We round A at random into the integers modulo m and look for a Sidon set there.

- The rounding respects every additive quadruple $a+b=c+d$ up to an error of one bucket, in any dimension.
- A code built from a planar difference set absorbs that error, so a Sidon set modulo m pulls back to one in A.
- Earlier arguments restrict to half of A at this step, and the code lets us skip that restriction.

03

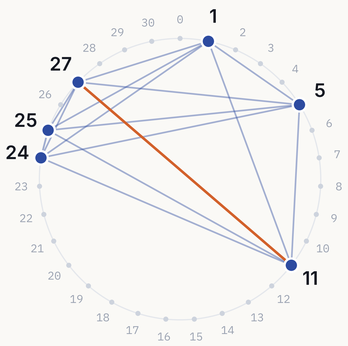
What we have so far

Every set of N numbers, and every N points in a torsion-free abelian group, has a Sidon subset of size about $0.385\sqrt{N}$, precisely $(\frac{2}{3\sqrt{3}} - o(1))\sqrt{N}$. That is twice the constant of Bailleul and Riblet.

In one dimension, with a random scale, two points land in the same bucket with probability exactly $1/m$.

04

A planar difference set



DIFFERENCES MOD 31

$a-b$	1	5	11	24	25	27
1	·	27	21	8	7	5
5	4	·	25	12	11	9
11	10	6	·	18	17	15
24	23	19	13	·	30	28
25	24	20	14	1	·	29
27	26	22	16	3	2	·

FIGURE 1. The residues 1, 5, 11, 24, 25 and 27 modulo 31 form a planar difference set: each of the 30 nonzero residues occurs exactly once as a difference of two of them, as the table shows (checked by computer). The highlighted chord gives the differences 15 and 16. Our codes are built from sets like this one.

05

Bounds on the constant

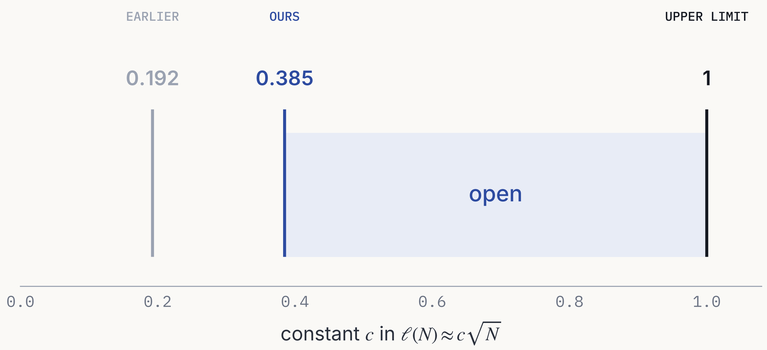
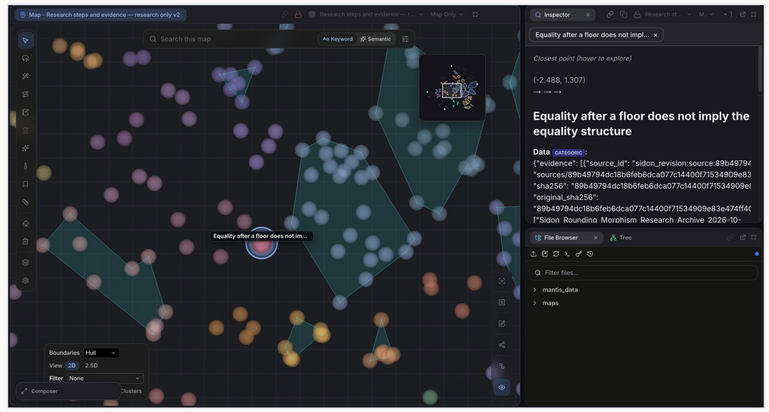


FIGURE 2. Our lower bound on c is twice the earlier one. The upper limit of 1 comes from $\{1, \dots, N\}$, whose largest Sidon subsets have about $\sqrt{N}$ elements.

06

In Mantis



632 RECORDS · THE OPEN RECORD

Equality after a floor does not imply the equality structure.

Each record in the project links to its source, as this one does.

ALSO ON RECORD

An earlier claim said that equality in the fractional capacity bound forces unit intervals. It was refuted on $\mathbb{T}_{14}$, where measure-one cells need not be intervals.

07

Open questions

- Arguments that use only $|A|$ and the collision law cannot pass $\frac{2}{3\sqrt{3}}$. What else about A could push the constant higher?
- Where between 0.385 and 1 does the true constant lie?